

# MATHEMATICAL METHODS (CAS)

VCE Units 3&4

# EXAM 1

12 Practice Examinations

2006–2011

A+ Practice Exams  
graduated difficulty with solutions



Solutions CD  
inside ▶

SAMPLE PAGES

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# MATHEMATICAL METHODS (CAS)

## EXAM 1 VCE Units 3 & 4

### Detailed Information

Test	Time (min)	Page	Key Skills and Knowledge
1	45	1	Functions and graphs. Transformations. Sketching and finding inverse functions. Absolute value functions. Exponential equations.
2	45	5	Polynomial functions. Transformations. Absolute value function. Addition of ordinates. Logarithmic and exponential functions. Product and composition of functions.
3	45	8	Exponential, logarithmic, circular and other functions. Transformations. The remainder theorem. Inverses. Sum and composition of functions.
4	45	11	Functions and graphs. Circular functions. Differentiation. Graphs of gradient functions. Rates of change and linear approximation.
5	60	14	Inverse and circular functions. Differentiation and applications including linear approximation and related rates. Absolute value functions.
6	60	18	Remainder theorem. Exponential and logarithmic equations. Applications of calculus. Areas under curves and between curves. Circular functions. Addition of ordinates. Absolute value functions.
7	60	22	Differentiation and applications. Exponential and circular functions. Area between 2 curves. Markov chains. Discrete random variables.
8	60	26	Circular functions. Exponential equations. Composition of functions. Logarithmic laws. Differential and integral calculus. Discrete random variables. Markov chains. Probability density functions. Normal distribution.
9	60	30	Absolute value functions. Logarithmic functions. Composition of functions. Applications of differential and integral calculus. Discrete random variables. Continuous random variables.
10	60	34	Product and composition of functions. Transformations. Graphs of gradient function. Solving trigonometric equations. Differentiation and its applications. Area under and between curves. Discrete and continuous probability distributions.
11	60	38	Sketching functions. Composition of functions. Logarithmic laws. Differentiation and its applications. Discrete and continuous probability distributions.
12	60	1	Bonus Detachable Examination. Functions and graphs. Inverses. Circular functions. Differential calculus and its application. Integral calculus and its applications. Discrete and continuous probability distributions.



Collectively, these 12 practice tests provide around 11 hours of structured revision questions for Mathematical Methods (CAS) written examination 1.

**Short answer and extended response questions**

**Specific instructions to students**

- Answer all of the questions in the spaces provided.

**QUESTION 1**

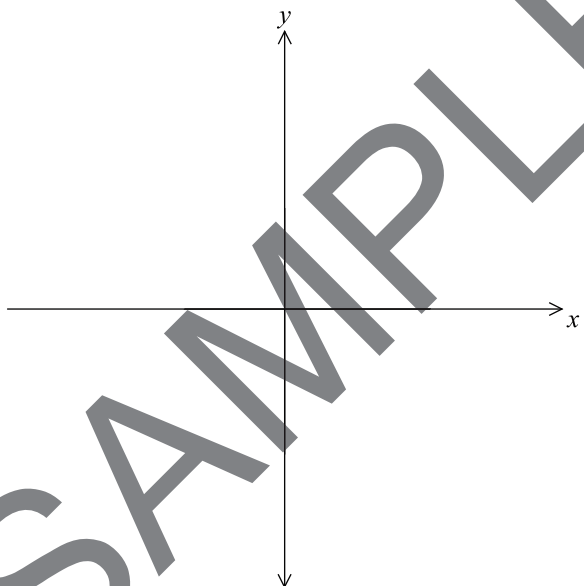
2 marks

Solve for  $x$ :  $\log_e(2x+1) - \log_e 3 = 3$

**QUESTION 2**

2 marks

Sketch  $y = x + |x|$ .



**QUESTION 3**

Total 2 marks

If  $f(x) = 1 + 3x^2$  and  $g(x) = \sqrt{x}$ , evaluate:

a  $f(g(4))$

1 mark

b  $g(f(4))$

1 mark

**QUESTION 4**

3 marks

Solve  $\cos\left(\frac{5\pi x}{12}\right) = -\frac{1}{2}$  for  $x \in \left[0, \frac{24}{5}\right]$ .

**QUESTION 5****Total 3 marks**Let  $f(x) = x \log_e x^2$ .**a** Find  $f'(x)$  and  $f'(1)$ .**1 mark****b** If  $x$  increases from 1 to  $1 + h$ , where  $h$  is a small amount, use your answers from **a** to find an approximate value for  $f(1 + h)$ .**2 marks****QUESTION 6****2 marks**The graph of the function  $y = \sqrt{x}$  is transformed as follows:

- A reflection in the  $y$ -axis
- A dilation of 3 units from the  $y$ -axis
- A translation of 1 unit parallel to the  $x$ -axis

Write down the equation of the rule of the transformed function.

**QUESTION 7****Total 3 marks**

Consider the following functions.

$$f: R \rightarrow R, f(x) = 2x - 1$$

$$g: R \rightarrow R, g(x) = 3kx + 1$$

**a** Define the rule for  $f(x)g(x)$ .**1 mark****b** Find the value of  $k$  for which  $f(x)g(x) = 0$  has exactly one solution.**2 marks****QUESTION 8****3 marks**The area of the region bounded by the curve  $y = mx^{\frac{1}{2}}$ , the  $x$ -axis and the line with equation  $x = 4$  is 20 square units. If  $m$  is a negative constant, find the value of  $m$ .

**QUESTION 9****3 marks**

A probability distribution is given below. Find the values of  $m$  and  $n$  if the mean of the distribution is 4.8.

$x$	0	2	3	7	$n$
$\Pr(X=x)$	0.1	0.3	$m$	0.3	0.2

**QUESTION 10****Total 4 marks**

A dog called Jaya is taken for a daily walk. The probability that she is obedient on a walk, given that there are no other dogs around, is 0.7, and the probability of her being obedient on a walk, given that there are other dogs around, is 0.6.

On a particular day, the probability that there are other dogs around is 0.3. Find the probability that on this particular day:

- a** Jaya is obedient on her walk

**2 marks**

- b** There are no other dogs around, given that Jaya is obedient.

**2 marks****QUESTION 11****Total 3 marks**

Students at a particular school attempted 3 different puzzles and their times (in seconds) are recorded below. The lower the time, the better the performance. The times are normally distributed.

	Mean	Standard deviation
Puzzle 1	150	10
Puzzle 2	200	12
Puzzle 3	80	5

Dottie did puzzle 1 in 154 seconds, puzzle 2 in 205 seconds and puzzle 3 in 83 seconds.

- a** Give Dottie's standardised time for each puzzle.

**2 marks**

- b** Hence state which puzzle Dottie performed best on.

**1 mark**

**QUESTION 12****Total 5 marks**

The contagious period for a certain virus is between 2 and 5 days after contact with the virus. The probability density function  $f(t) = \frac{2}{9}(t-2)(5-t)$  gives the probability that symptoms will appear after  $t$  days.

- a** Sketch the graph. On the graph, shade the area to show the probability that the symptoms will appear within 3 days after contact with the virus. **2 marks**

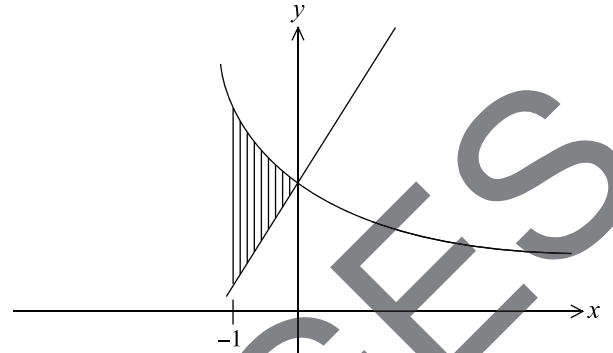


- b** Find the exact probability that the symptoms will appear within 3 days after contact with the virus. **2 marks**

- c** Find the mode. **1 mark**

**QUESTION 13****Total 5 marks**

The graph of  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = e^{-2x} + 2$  is shown. The normal to the graph of  $f$  where it crosses the  $y$ -axis is also shown.



- a** Find the equation of the normal to the graph of  $f$  where it crosses the  $y$ -axis. **2 marks**

- b** Find the exact area of the shaded region between  $x = -1$  and  $x = 0$ . **3 marks**